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Performance of Variance Targeting Estimator (VTE) under Misspecified Error Distribution Assumption

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ABSTRACT

Parameter estimation in Generalized Autoregressive Conditional Heteroscedastic (GARCH) model has received much attention in the literature. Commonly used quasi maximum likelihood estimator (QMLE) may not be suitable if the model is misspecified. Alternatively, we can consider using variance targeting estimator (VTE) as it seems to be a better fit for misspecified initial parameters. This paper extends the application to see how both QMLE and VTE perform under error distribution misspecifications. Data are simulated under two error distribution conditions: one is to have a true normal error distribution and the other is to have a true student-t error distribution with degree of freedom equals to 3. The error distribution assumption that has been selected for this study are: normal distribution, student-t distribution, skewed normal distribution and skewed student-t. In addition, this study also includes the effect of initial parameter specification. The analyses are divided into two case designs. Case 1 $\omega_0 = 0.1, \alpha_0 = 0.05, \beta_0 = 0.85$ is when to represent the well specified initial parameters while Case 2 is when $\omega_0 = 1, \alpha_0 = 0, \beta_0 = 0$ to represent misspecified initial parameters. The results show that both QMLE and VTE estimator performances for misspecified initial parameters may not improve in well specified error distribution assumptions. Nevertheless, VTE shows a favourable performance compared to QMLE when the error distribution assumption is not the same as true underlying error distribution.

Keywords: GARCH, variance targeting, parameter estimation, error distribution

INTRODUCTION

Autoregressive conditional heteroscedastic (ARCH) model was first introduced by Engle (1982) and later extended as generalized ARCH (GARCH) by Bollerslev (1986). Its ability to mirror clustering characteristic in financial data makes this model popular, especially GARCH model for modelling and

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forecasting financial market volatility. Clustering means high volatility tends to be followed by large changes, of either positive or negative signs or vice versa as mentioned by Mandelbrot (1963). This model is important because it has important roles in several financial applications such as option pricing, asset allocation and hedging.

Many studies have discussed the ability of the GARCH model in forecasting volatility. Poon and Granger (2003) have conducted an extensive review of the GARCH model but their findings were inconclusive which could have been due to the different sample period, sample frequency, forecast horizon, loss function and the proxy used for the ex post variance (Wilhelmsson, 2006). As the matter of fact, our study investigate further on these specifications issue as well as the error distribution assumption selection.

In discussing the GARCH model, two types of distribution have to be considered: the marginal, also known as the unconditional distribution, and the conditional distribution. As the GARCH model itself depends on time, it is known that the conditional error distribution has to follow a certain distribution even though the true underlying error distribution is naturally unknown. As a result, practitioners always assume the financial datasets under study to follow certain available distribution. The most preferable assumption is that the financial data follows a normal distribution because the commonly used estimator, which is a quasi-maximum likelihood estimator (QMLE), works well under such conditions. However, the financial data series is actually not normally distributed as shown by Mikosch and Stărică (2004), thus motivating our study to address this issue.

Among the studies that have discussed about the error distribution assumption specification are Hamilton and Susmel (1994), Franses and Ghijsels (1999), Lopez (2001), and Wilhelmsson (2006). Hamilton and Susmel (1994) applied Markov switching GARCH model and allowed the error term to be distributed according to a normal, Student's-t or generalized error distribution. By using weekly stock market data, it is found that the GARCH model with a Student's-t distribution performs best, followed by the generalized error distribution when the forecast performance of the one-week horizon is evaluated. On the other hand, Franses and Ghijsels (1999) drew different conclusion when using weekly European stock market data to evaluate the out-of-sample forecast. It is found that the GARCH model with t-distributed error is the worst model. Lopez (2001) checked the performance if GARCH(1,1) model fitted with the normal, Student's-t and generalized error distribution on four daily exchange rate series. It is shown that the performance of the models in-sample and out-of-sample are different and highlighting the importance of out of sample results as a model selection criteria. The results are mixed depending on the data series and loss function. Meanwhile, Wilhelmsson (2006) found that when the models were estimated, allowing for a skewed and excess kurtosis to be taken into account, it improved the log-likelihood. On the other hand, the out-of-sample results showed that allowing for skewness does not lead to any improvement over the normal distribution.

Mixed results, as the above example, might be due to several possible specifications that can be applied to the GARCH model which may lead to misspecification problem. One particular aspect of error misspecification impact is it will reduce the estimator performance for GARCH model parameters. Engle and Gonzalez-Rivera (1991) found that QMLE can suffered a 84% loss of efficiency due to misspecification of the error density. There is one particular estimator that could provide a good model even though there might be a misspecification which is variance targeting estimator (VTE) by Engle and Mezrich (1996). The VTE is a two-step estimator based on reparameterization of the volatility equation where the intercept is replaced by the returns unconditional variance.

Francq et al. (2009) detailed the asymptotic properties of the VTE and list the method's advantages and disadvantages by studying the performance of VTE and QMLE toward modelling the simple univariate GARCH(1,1). Besides numerical simplicity, it is found that VTE can ensure the estimated unconditional variance of the GARCH model is equal to the sample variance. Hence, it is possible that in case of misspecification when the true underlying process is not GARCH, it can provide a superior result than QMLE (Francq et al., 2009). It is useful for prediction over long horizons. The main drawback of VTE is it needs a finite fourth order moment to retain its efficiency. Vaynman and Beare (2014) confirm it by investigating the VTE performance under infinite fourth moment with a heavy tailed distribution. It is found that in heavier tail condition, the finite fourth order moment is likely to be infinite and concluded that VTE should be used with caution in application when the distribution is heavy tailed.

The VTE may serve as a good alternative to QMLE especially because of its robustness towards model misspecification and seems to ease the numerical process. The main focus of this study is the application of VTE toward error distribution assumption misspecification, addressing gap in the literature. It has to be noted that this study is limited to in-sample model fitted to evaluate how well VTE performs in this scenario.

MODEL

This section presents GARCH (1,1) parameter estimation using two different estimators which are QMLE and VTE. GARCH(1,1), QMLE and VTE are explained as below;

Univariate GARCH

For GARCH (1,1), the model that has been used in this research is expressed as:

$$\text{GARCH (1,1)} = \begin{cases} \epsilon_t = \sqrt{h_t}\eta_t \\ h_t = \omega + \alpha_0\epsilon_{t-1}^2 + \beta_0 h_{t-1} \end{cases} \tag{1}$$

where η_t is a sequence of iid with unit variance, $\omega > 0$, $\alpha_0 \geq 0$, $\beta_0 \geq 0$ and $\alpha_0 + \beta_0 < 1$.

QMLE Estimation

The asymptotic behaviour of QMLE is the reason why. Thus, it is important to use QMLE as a benchmark to compare the performance of proposed estimator technique. Using GARCH (1,1), the estimators which need to be estimated are $\theta_0 = (\omega, \alpha, \beta)'$. QMLE under assumption of derived as any measurable solution $\hat{\theta}_n$ of $\hat{\theta}_n = \arg\max_{0\in\Theta} L(\theta)$ of where

$$L_n(\theta) = L_n(\theta\colon \epsilon_1, \ldots, \epsilon_n) = \prod_{t=11}^{n} \frac{1}{\sqrt{2\pi\tilde{\sigma}_t^2}} \exp(-\frac{\epsilon}{2\tilde{\sigma}_t^2}) \tag{2}$$

$\tilde{\sigma}_t^2$ defined as; $\tilde{\sigma}_t^2 = \tilde{\sigma}_t^2(\theta) = \omega + \alpha_0 \epsilon_{t-1}^2 + \beta_0 \tilde{\sigma}_{t-1}^2$

Thus, implementing the logarithm gives the maximising of the likelihood equal to maximising $\tilde{I}_n$ with respect to θ,
where,

$\tilde{I}_n(\theta) = \frac{1}{2n}\sum_{t=1}^{n} \tilde{\ell}_t$ where, $\tilde{\ell}_t = \tilde{\ell}_t(\theta) = -\left(\frac{\epsilon_t^2}{\tilde{\sigma}_t^2} + \ln \tilde{\sigma}_t^2\right)$

VTE Estimation

The VTE consists of two steps. First, the unconditional variance of the observed data is estimated by a moment estimator and next, the remaining parameters are estimated by the QMLE. The steps are explained below.

Consider GARCH (1,1) as (1) where $\theta_0 = (\omega_0, \alpha_0, \beta_0)'$ is an unknown parameter (η_t) and a sequence of independent in variance of identically distributed (i.i.d) random variables such that $E\eta_t^2 = 1$ and $\omega_0 > 0$, $\alpha_0 > 0$, $\beta_0 \geq 0$. Under condition $\alpha_0 + \beta_0 < 1$, this model admits a second-order stationary solution, which the unconditional variance is given by,

$$\gamma_0 := \sigma^2(\omega_0, \alpha_0, \beta_0) = \frac{\omega_0}{1-\alpha_0-\beta_0} := \frac{\omega_0}{\kappa_0}$$

A reparameterisation of the model with $\vartheta_0 = (\gamma_0, \alpha_0, \kappa_0)'$ yields;

$$\epsilon_t = \sqrt{h_t}\eta_t,$$
$$h_t = h_{t-1} + \kappa_0(\gamma_0 - h_{t-1}) + \alpha_0(\epsilon_{t-1}^2 - h_{t-1})$$

and κ_0 is the speed of mean reversion.

Writing (6) as $h_t = \kappa_0\gamma_0 + \alpha_0\epsilon_{t-1}^2 + \beta_0 h_{t-1}$ where $\kappa_0 + \alpha_0 + \beta_0 = 1$, we can interpret the volatility h_t at time t as weighted average of the long-run variance γ_0, the square of the last return ϵ_{t-1}^2 and the previous volatility h_{t-1}. In this average, κ_0 is the weight of the long-run variance. This reparemetrisation limits; $\kappa_0, \gamma_0 > 0$, $\alpha_0 \geq 0$ and $\kappa_0 + \alpha_0 \leq 1$.

Let $(\epsilon_1, \ldots, \epsilon_n)$ be a realisation of length n of the unique nonacticipative second-order stationary solution (ϵ_t) to model (1). In this framework, VTE involves (i) reparametrising the model as in (6), and (ii) estimating γ_0 by the sample variance using moment estimator and then $\lambda_0 := (\alpha_0, \kappa_0)'$ by the QMLE. The QMLE of θ_0 is denoted by $\hat{\theta}_n^* := (\hat{\omega}_n^*, \hat{\alpha}_n^*, \hat{\beta}_n^*)'$. Two consistent estimator of λ_0 are the sample variance and the QML-based estimator given by $\hat{\sigma}_n^2 = \frac{1}{n}\sum_{t=1}^{n} \epsilon_t^2$ (7) and $\sigma^2(\hat{\theta}_n^*) = \frac{\hat{\omega}_n^*}{1-\hat{\alpha}_n^*-\hat{\beta}_n^*}$.

Consider a parameter space $\Lambda \subset \{(\alpha, \kappa) | \alpha \geq 0, \kappa > 0, \alpha + \kappa \leq 1\}$. All the vectors are considered as column vectors written as row vectors. In particular, we write $\vartheta_0 = (\gamma_0, \lambda'_0)'$ and at the point $\vartheta = (\gamma, \lambda) \in (0, \infty) \times \Lambda$, the QMLE of the sample given by

$$\tilde{L}_n(\vartheta) = \tilde{L}_n(\gamma, \lambda) = \prod_{t=1}^{n} \frac{1}{\sqrt{2\pi\tilde{\sigma}_t^2(\vartheta)}} \exp\{-\frac{\epsilon_t^2}{2\tilde{\sigma}_t^2(\vartheta)}\} \qquad (3)$$

where the $\tilde{\sigma}_t^2(\vartheta)$ are defined as recursively, for $t \geq 1$, by $\tilde{\sigma}_t^2(\vartheta) = \kappa\gamma + \alpha\epsilon_{t-1}^2 + (1-\kappa-\alpha)\tilde{\sigma}_t^2(\vartheta)$ with the initial values ϵ_0 and $\tilde{\sigma}_t^2(\vartheta) := \sigma_0^2$. Since the parameter λ_0 is estimated by the sample variance $\hat{\sigma}_n^2$, the variance targeting version of the QMLE function is

$$L(\lambda) = L(\hat{\sigma}_n^2, \lambda) = \prod_{t=1}^{n} \frac{1}{\sqrt{2\pi\sigma_{t,n}^2}} \exp\{-\frac{\epsilon_t^2}{2\sigma_{t,n}^2}\} \quad (4)$$

where the $\sigma_{t,n}^2 := \sigma_{t,n}^2(\lambda) = \kappa_0\hat{\sigma}_n^2 + \alpha_0\epsilon_{t-1}^2 + (1-\kappa-\alpha)\hat{\sigma}_{t,n}^2$ with $\sigma_{0,n}^2 = \sigma_0^2$. A VTE of λ_0 is defined as any measurable solution $\tilde{\lambda}_n$ of;

$$\lambda = \arg\max_{\lambda\in\Lambda} L_n(\lambda) = \arg\min_{\lambda\in\Lambda} \tilde{I}_n(\lambda)$$

and

$$\tilde{I}_n(\lambda) = \frac{1}{2n}\sum_{t=1}^{n} \ell_{t,n}, \ell_{t,n} := \ell_{t,n}(\lambda) = -(\frac{\epsilon_t^2}{\sigma_{t,n}^2} + \ln \sigma_{t,n}^2) \quad (5)$$

METHODOLOGY

A simulation study is considered to investigate the performance of the two estimators, QMLE and VTE.

First, random variable of ε_t is simulated under two different underlying error distributions, one with normal distribution that has a finite moment across all orders and the other under the Student's-t distribution with degrees of freedom equal to 3 to represent the infinite fourth order moment. Datasets are simulated into three different sample size, in which n=500, 1000 and 5000 for both true error distribution setup.

Next, using "rugarch" package in R-programming, different error distribution assumption is used to model GARCH (1,1). The distribution assumption is normal, Student-t, skewed normal and skewed Student's-t. Skewed distributions for both normal and Student's-t are considered to represent the assumption of a heavy tailed error.

Two case designs, Case 1 and Case 2, are established to differentiate initial parameter specifications. Case 1 is when $\omega_0 = 0.1, \alpha_0 = 0.05, \beta_0 = 0.85$ are used to represent the well specified initial parameters while Case 2 is when $\omega_0 = 1, \alpha_0 = 0, \beta_0 = 0$ are used as the representative of misspecified initial parameters.

RESULTS AND DISCUSSION

For Case 1 (well specified initial parameters), both QMLE and VTE under well specified error distribution (error distribution assumption is the same with the true underlying error distribution) outperform other distribution assumption setting. This result applies if the true distribution is normally distributed. At the same time as shown in Table 1 VTE perform better than QMLE if the error distribution assumption is misspecified. However, the result is different if the simulated data true underlying error is Student-t distributed. It seems that a well specified error distribution does not help in improving the likelihood values for this scenario. Estimators under normal and skewed normal assumption perform better than Student-t and skewed Student-t based on Table 2. Comparing the results in Table 1 and 2, we can conclude that the performance of both QMLE and VTE is reduced when the true underlying distribution is heavy-tailed.

For Case 2, QMLE and VTE under misspecification error distribution assumption (error distribution assumption is different with the true underlying error distribution) perform better than when the estimators are under well specified condition for both true error distributions (refer Table 3 and 4.). The VTE only outperforms QMLE when n=500 and n=5000.

Based on the results, there are several important findings as well. One of it is, VTE needs less processing time than QMLE. Furthermore, for Case 1, the sample size must be greater than 1000 to produce the significant parameters for all. The significance value must be less than 0.05 in order to conclude the significance of the parameter.

Under infinite fourth order moment, as suspected, the VTE performance is reduced. The QMLE outperforms most of the VTE under all levels if we compare the likelihood and standard error produce for each parameter. But still, VTE produces more significant parameters than QMLE.

Table 1
Parameters estimation under true normal error distribution for Case 1

Sample		**n=500**					**n=1000**					**n=5000**				
Distribution Assumption		ω	α	β	Likelihood	Times	ω	α	β	Likelihood	Times	ω	α	β	Likelihood	Times
Normal	QMLE	0.2882*	0.0327*	0.9391	-1282.75	0.11	0.6941	0.0788	0.8554	-2577.31	0.15	1.1443	0.0589	0.8242	-12761.09	0.70
		0.2843	**0.0190**	**0.0407**			**0.2978**	**0.0211**	**0.0406**			**0.3609**	**0.0121**	**0.0452**		
	VTE	0.2893	0.0323*	0.9389	-1282.76	0.08	0.6940	0.0784	0.8555	-2577.31	0.08	1.1437	0.0588	0.8242	-12761.09	0.23
			0.0687	**0.1503**				**0.0195**	**0.0404**				**0.0118**	**0.0452**		
Student-t	QMLE	0.2914*	0.0322*	0.9393	-1282.66	0.30	0.6861	0.0785	0.8566	-2577.23	0.48	1.1189	0.0582	0.8277	-12760.41	1.84
		0.2941	**0.0192**	**0.0415**			**0.3003**	**0.0213**	**0.0409**			**0.3707**	**0.0124**	**0.0465**		
	VTE	0.2929	0.0319*	0.9390	-1282.67	0.21	0.6865	0.0781	0.8565	-2577.23	0.31	1.0880	0.0568	0.8319	-12760.42	0.86
			0.0498	**0.1105**				**0.0001**	**0.0406**				**0.0120**	**0.0456**		
Skewed-Normal	QMLE	0.2950*	0.0328*	0.9382	-1282.64	0.14	0.6700	0.0779	0.8588	-2576.82	0.19	1.1527	0.0592	0.8231	-12760.99	0.76
		0.2923	**0.0193**	**0.0418**			**0.2907**	**0.0209**	**0.0399**			**0.3608**	**0.0121**	**0.0452**		
	VTE	0.2958	0.0324*	0.9381	-1282.65	0.08	0.6699	0.0773	0.8589	-2576.82	0.11	1.1520	0.0590	0.8231	-12760.99	0.44
			0.0414	**0.0919**				**0.0192**	**0.0394**				**0.0118**	**0.0452**		
Skewed-Student	QMLE	0.2969*	0.0324*	0.9385	-1282.57	0.58	0.6615	0.0777	0.8599	-2576.77	0.68	1.1262	0.0584	0.8266	-12760.25	2.41
		0.2998	**0.0195**	**0.0424**			**0.2935**	**0.0212**	**0.0403**			**0.3687**	**0.0124**	**0.0463**		
	VTE	0.2981	0.0321*	0.9382	-1282.58	0.24	0.6620	0.0770	0.8599	-2576.78	0.40	1.1297	0.0582	0.8262	-12760.26	1.61
			0.0367	**0.0820**				**0.0194**	**0.0398**				**0.0121**	**0.0465**		

*Not Significant (p-value>0.05)

Table 2
Parameters estimation under true student's-t (df=3) error distribution for Case 1

Sample Distribution Assumption		n=500 ω	α	β	Likelihood	Times	n=1000 ω	α	β	Likelihood	Times	n=5000 ω	α	β	Likelihood	Times
Normal	QMLE	0.0033*	0.0024*	0.9965	-1305.17	0.14	1.3205*	0.0651	0.8010	-2558.57	0.20	0.6506	0.0349	0.8980	-12758.33	0.46
		0.0033	**0.0026**	**0.0003**			**0.6887**	**0.0249**	**0.0849**			**0.1855**	**0.0078**	**0.0237**		
	VTE	5.9337	0.0937*	0.3624*	-1301.62	0.10	1.3255	0.0658	0.8008	-2558.57	0.09	0.6505	0.0349	0.8981	-12758.33	0.21
			0.0497	**0.4605**				**0.0245**	**0.0851**				**0.0073**	**0.0228**		
Student-t	QMLE	0.0047*	0.0017*	0.9972	-1301.24	0.19	1.3090*	0.0659	0.8018	-2558.95	0.39	0.6527	0.0351	0.8977	-12758.02	1.69
		0.0289	**0.0028**	**0.0002**			**0.6901**	**0.0254**	**0.0851**			**0.1888**	**0.0079**	**0.0241**		
	VTE	5.5263	0.0899*	0.4034*	-1298.64	0.15	1.3112	0.0663	0.8018	-2558.95	0.35	0.6511	0.0351	0.8979	-12757.99	1.12
			0.0544	**0.5292**				**0.0075**	**0.0851**				**0.0076**	**0.0236**		
Skewed-Normal	QMLE	0.0034*	0.0022*	0.9967	-1305.10	0.15	1.3216*	0.0653	0.8007	-2558.56	0.27	0.6509	0.0350	0.8980	-12758.32	0.75
		0.0258	**0.0025**	**0.0002**			**0.6907**	**0.0251**	**0.0852**			**0.1857**	**0.0078**	**0.0237**		
	VTE	6.1949	0.0957*	0.3364*	-1301.50	0.13	1.3261	0.0660	0.8006	-2558.57	0.12	0.6508	0.0349	0.8980	-12758.32	0.28
			0.0517	**0.5142**				**0.0246**	**0.0853**				**0.0074**	**0.0231**		
Skewed-Student	QMLE	0.0047*	0.0016*	0.9973	-1301.23	0.26	1.3046*	0.0667	0.8019	-2559.25	0.49	0.6525	0.0352	0.8977	-12758.15	2.44
		0.0289	**0.0028**	**0.0002**			**0.6954**	**0.0259**	**0.0857**			**0.1896**	**0.0080**	**0.0242**		
	VTE	5.6735	0.0908*	0.3891*	-1298.62	0.21	1.3031	0.0668	0.8020	-2559.25	0.41	0.6522	0.0351	0.8977	-12758.15	2.06
			0.0555	**0.5640**				**0.0251**	**0.0855**				**0.0075**	**0.0232**		

*Not Significant (p-value>0.05)

Table 3
Parameters estimation under true normal error distribution for Case 2

Sample Distribution Assumption		n=500					n=1000					n=5000				
		ω	α	β	Likelihood	Times	ω	α	β	Likelihood	Times	ω	α	β	Likelihood	Times
Normal	QMLE	0.0009	0.0000*	0.9989	-714.24	0.13	0.0036	0.0000*	0.9965	-1440.90	0.23	0.0234	0.0070	0.9690	-7111.04	0.50
		0.0001	**0.0000**	**0.0001**			**0.0003**	**0.0000**	**0.0003**			**0.0033**	**0.0029**	**0.0000**		
	VTE	0.1090	0.0000*	0.8930*	-714.31	0.08	0.0612	0.0107	0.9306	-1440.46	0.08	0.0236	0.0071	0.9694	-7111.04	0.37
			0.0002	**2.4242**				**0.0140**	**0.1030**				**0.0033**	**0.0009**		
Student-t	QMLE	0.0009	0.0000*	0.9989	-714.42	0.20	0.0010	0.0000*	0.9989	-1441.13	0.34	0.0240	0.0070	0.9690	-7112.29	1.31
		0.0001	**0.0000**	**0.0001**			**0.0001**	**0.0000**	**0.0001**			**0.0033**	**0.0032**	**0.0009**		
	VTE	0.0191	0.0000*	0.9812	-714.48	0.17	0.0619	0.0109*	0.9297	-1440.71	0.28	0.0240	0.0071	0.9689	-7112.37	0.98
			0.0000	**0.0005**				**0.0143**	**0.1054**				**0.0032**	**0.0002**		
Skewed-Normal	QMLE	0.0009	0.0000*	0.9989	-713.83	0.16	0.0594*	0.0104*	0.9324	-1440.27	0.34	0.0230	0.0070	0.9699	-7110.90	0.72
		0.0001	**0.0000**	**0.0001**			**0.0707**	**0.0130**	**0.0729**			**0.0029**	**0.0029**	**0.0000**		
	VTE	0.3874	0.0131*	0.6068*	-713.83	0.11	0.0601	0.0105*	0.9319	-1440.27	0.09	0.0232	0.0071	0.9697	-7110.91	0.31
			0.0382	**0.6276**				**0.0140**	**0.1059**				**0.0032**	**0.0009**		
Skewed-Student	QMLE	0.0009	0.0000*	0.9989	-714.23	0.31	0.0010	0.0000*	0.9989	-1441.19	0.58	0.0242	0.0071	0.9688	-7113.37	2.23
		0.0001	**0.0001**	**0.0001**			**0.0001**	**0.0001**	**0.0001**			**0.0034**	**0.0033**	**0.0009**		
	VTE	0.0397	0.0000*	0.9610	-714.28	0.28	0.0612	0.0109	0.9304	-1440.78	0.37	0.0241	0.0071	0.9688	-7113.37	1.29
			0.0000	**0.0002**				**0.0146**	**0.1097**				**0.0034**	**0.0009**		

*Not Significant (p-value>0.05)

Table 4

Parameters estimation under true student's-t (df=3) error distribution for Case 2

Sample Distribution Assumption		n=500					n=1000					n=5000				
		ω	α	β	Likelihood	Times	ω	α	β	Likelihood	Times	ω	α	β	Likelihood	Times
Normal	QMLE	0.0086	0.0000*	0.9990	-1086.07	0.24	0.3944	0.0324	0.8109	-1872.88	0.32	0.1258*	0.0094	0.9449	-9633.68	0.64
		0.0019	**0.0000**	**0.0007**			**0.0975**	**0.0124**	**0.0430**			**0.0749**	**0.0028**	**0.0310**		
	VTE	0.5473	0.0000*	0.8816*	-1092.39	0.10	0.3954	0.0326	0.8108	-1872.89	0.09	0.1267	0.0095	0.9447	-9633.70	0.23
			0.0000	**28.2311**				**0.0119**	**0.0430**				**0.0018**	**0.0113**		
Student-t	QMLE	Na	Na	Na	Na	Na	0.0118	0.0000*	0.9957	-1726.82	0.26	0.0533	0.0014*	0.9824	-8915.99	1.31
							0.0020	**0.0000**	**0.0003**			**0.0064**	**0.0017**	**0.0001**		
	VTE	0.5866	0.0000*	0.8732	-870.87	0.14	0.4142	0.0148*	0.8213	-1726.01	0.14	0.0365	0.0010	0.9857	-8918.30	0.61
			0.0000	**0.1551**				**0.0142**	**0.1149**				**0.0003**	**0.0000**		
Skewed-Normal	QMLE	0.0061	0.0000*		-1069.67	0.22	0.0232	0.0020*	0.9889	-1878.72	0.21	0.1123	0.0095	0.9496	-9628.03	0.84
		0.0004	**0.0000**				**0.0051**	**0.0017**	**0.0007**			**0.0219**	**0.0026**	**0.0088**		
	VTE	Na	Na	Na	Na	Na	0.3820	0.0322	0.8166	-1870.52	0.10	0.1132	0.0097	0.9494	-9628.07	0.30
								0.0119	**0.0419**				**0.0003**	**0.0060**		
Skewed-Student	QMLE	0.0439	0.0000*	0.9811	-867.38	0.27	0.0129	0.0000*	0.9954	-1726.44	0.37	0.0529	0.0015*	0.9824	-8915.64	2.03
		0.0042	**0.0000**	**0.0034**			**0.0022**	**0.0000**	**0.0002**			**0.0064**	**0.0017**	**0.0001**		
	VTE	0.8649	0.0000*	0.8130*	-870.86	0.18	0.4039	0.0142*	0.8258	-1725.63	0.20	0.0368	0.0010	0.9856	-8917.96	1.05
			0.0000	**10.7712**				**0.0138**	**0.1133**				**0.0003**	**0.0000**		

*Not Significant (p-value>0.05)

CONCLUSION

In conclusion, VTE shows promising performance when dealing with the misspecification model for both initial parameters and error distribution. But under misspecified initial parameters, a well specified error distribution assumption does not fix the estimators performance suggesting that these two factors are to be treated differently, but more evidences are needed to arrive at this conclusion. Future research should examine the performance of these two estimators in managing volatility forecasting in the presence of leverage effect. Asymmetric GARCH can also be used in future research. More research is needed on parameter estimation so that the most efficient model can be built and helping in reducing the risk faced in financial data series. In addition, out-of-sample forecast evaluation of real datasets might help in finding more conclusive evidence of VTE effectiveness.

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